State Space Representation of the Cobb-Douglas Production Function: Time Variation of Total Factor Productivities (TFPs) in the 47 prefectures of Japan

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Introduction

The Cobb-Douglas production function (CDPF) is a classical function used to predict economic production. The CDPF is usually written as the log-linear regression model with two constant parameters: total factor productivity (TFP) and capital elasticity (or labor elasticity). However, the assumption of constant TFP has no scientific evidence and can lead to unrealistic prediction results. This study extends the CDPF to a state space model (SSM) with dynamic parameters and estimates time variation of TFPs in the 47 prefectures of Japan.

Data

- The data period is 1990—2014 (N = 25).
- Economic value is measured by JPY in 2005.
- Economic production, gross fixed capital formation, and labor population were calculated from Prefectural Accounts (Cabinet Office of Japan).
- Fixed capital stock was estimated by the perpetual inventory method (depreciation rate 5%). The value in 1989 was taken from Commercial Capital Stock by Prefecture (Cabinet Office of Japan).

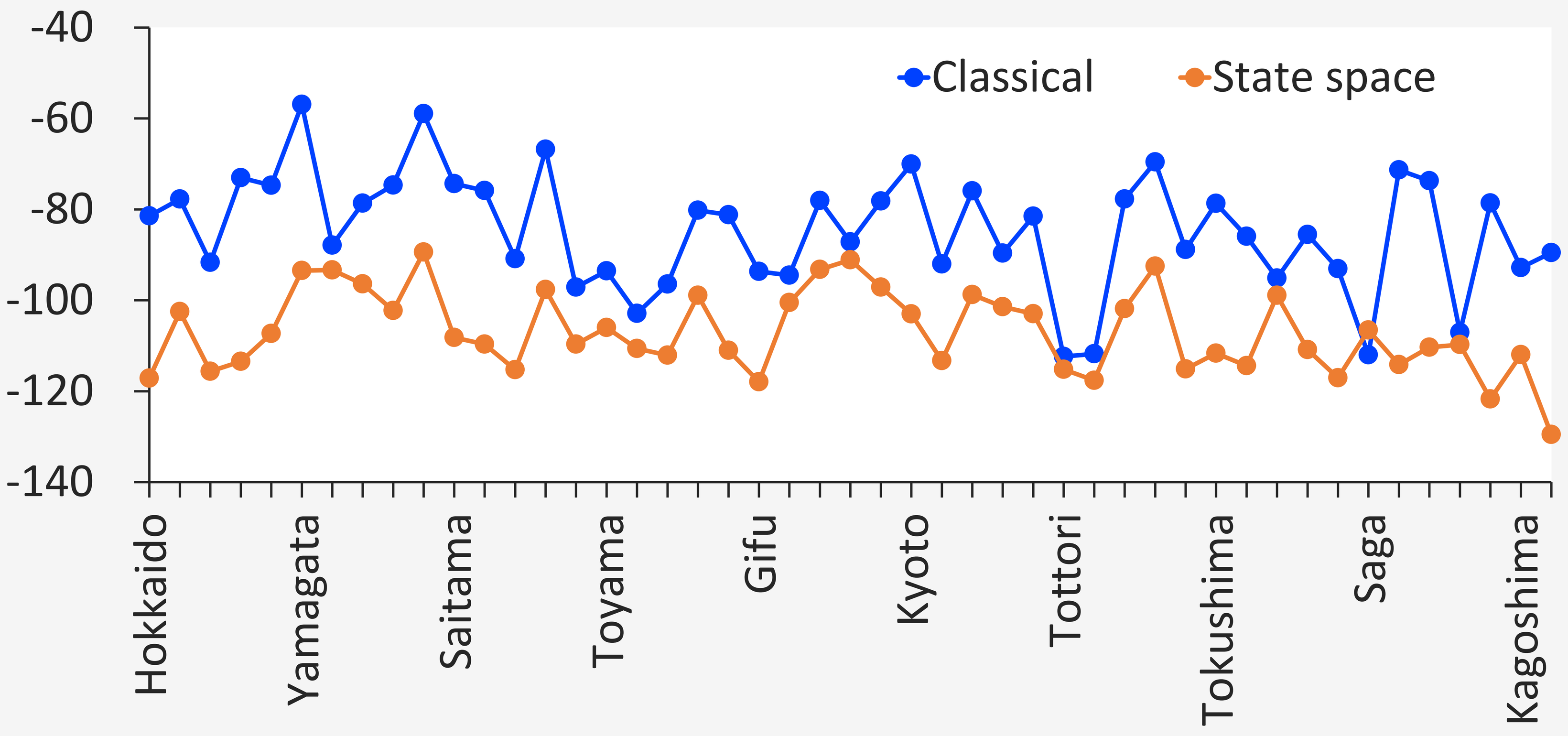

Fig 1: AIC of the CDPFs for the 47 prefectures of Japan.
Note: AIC is given by $\mathrm{AIC} = -2L + 2(p+q)$. L is the maximum log likelihood, p is the number of unknown error variances, and q is the number of unknown coefficients (including the intercept).

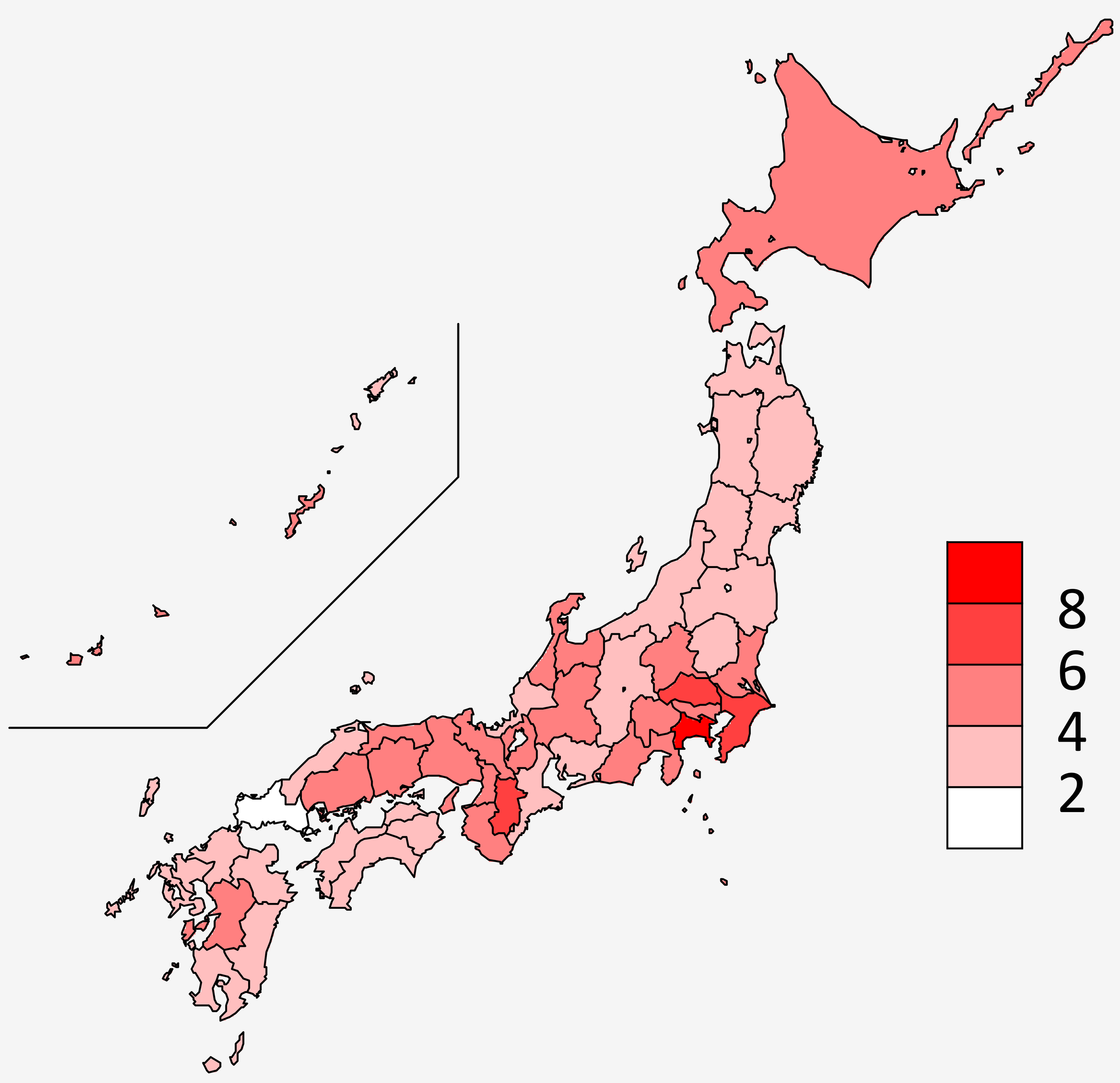

Fig 2: TFP estimates in the 47 prefectures of Japan in 2014.

Model

➢ Classical CDPF

$$\log Y_t = \log A + \theta \log K_t + (1-\theta)\log L_t + v_t, \quad v_t \sim N(0, V)$$

Y_t: Economic production A: Constant TFP K_t: Fixed capital stock
L_t: Labor population θ: Capital elasticity v_t: Observation error
V: Observation error variance

- Parameters are estimated by the OLS.

➢ State Space Representation of the CDPF

$$\log Y_t = \log A_t + \theta \log K_t + (1-\theta)\log L_t + v_t, \quad v_t \sim N(0, V)$$
$$\log A_{t+1} = \log A_t + w_t, \quad w_t \sim N(0, W)$$

A_t: Time-varying TFP w_t: State error W: State error variance

- We assume that the TFP follows Gaussian random walk.
- Parameters are estimated by the MLE and Kalman filter.
- There are R packages for state space modeling (e.g. dlm and KFAS).

Results

- In every prefecture except for Saga, the AIC of the SSM was lower than the AIC of the classical model (**Fig 1**). This result supports the hypothesis that the TFP is stochastically time varying.
- According to the SSMs, capital elasticity ranged from 0.017 in Kanagawa to 0.474 in Yamaguchi (mean 0.219, SD 0.082).
- The TFP in 2014 ranged from 1.890 in Yamaguchi to 9.073 in Kanagawa (mean 4.193, SD 1.327) (**Fig 2**).
- Tokyo ranked first in the size of economic production but ranked ninth in the TFP. The development of Tokyo is attributed to the concentration of fixed capital stock and labor population rather than high production efficiency.
- The TFP growth rate between 1990 and 2014 ranged from -5.0% in Saga to +20.2% in Yamagata (**Fig 3**). Although various new technologies were developed during the period, the TFP growth rate was less than 10% in 35 prefectures.
- The estimates of state error variances can be used for evaluating the uncertainty of the TFPs in the future.

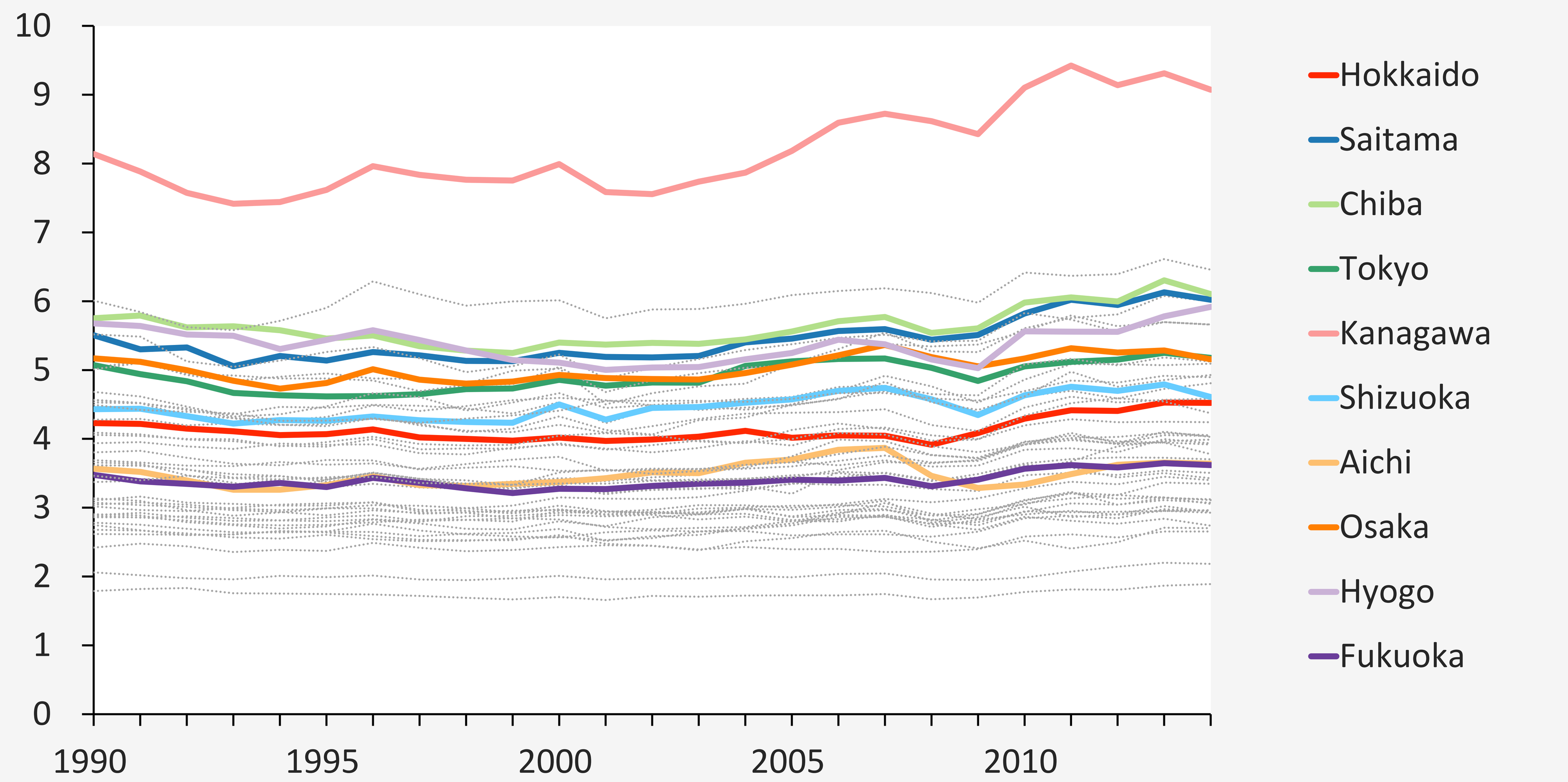

Fig 3: Time variation of the TFPs in the 47 prefectures of Japan, 1990—2014.
Note: The solid lines indicate the top 10 prefectures in economic production.